Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

MATHEMATICS **9709/13**

Paper 1 Pure Mathematics 1 **May/June 2024**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

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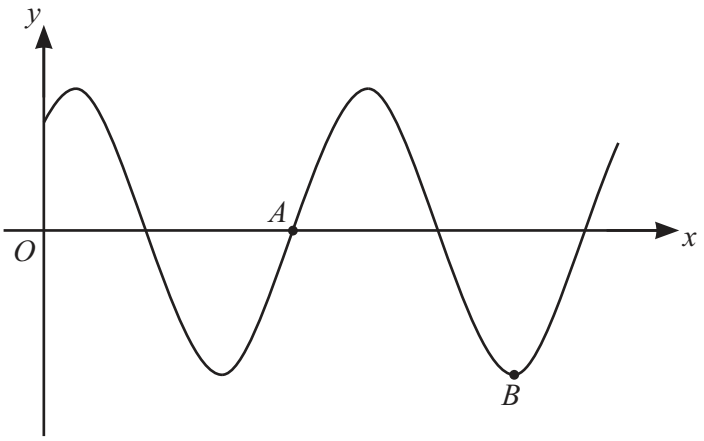
1 Find the coefficient of x^2 in the expansion of

$$(2-5x)(1+3x)^{10}.$$ [4]

2 (a)



The diagram shows the curve $y = k\cos(x - \frac{1}{6}\pi)$ where k is a positive constant and x is measured in radians. The curve crosses the x-axis at point A and B is a minimum point.

Find the coordinates of A and B. [3]

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(b) Find the exact value of t that satisfies the equation

$$3\sin^{-1}(3t) + 2\cos^{-1}\left(\tfrac{1}{2}\sqrt{2}\right) = \pi. \quad [2]$$

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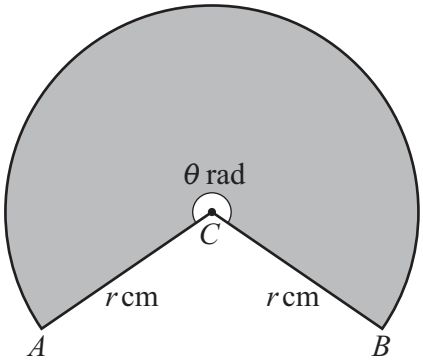

The diagram shows a sector of a circle with centre C. The radii CA and CB each have length r cm and the size of the reflex angle ACB is θ radians. The sector, shaded in the diagram, has a perimeter of 65 cm and an area of 225 cm^2.

(a) Find the values of r and θ. [4]

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(b) Find the area of triangle ACB. [2]

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4 **(a)** Show that the equation $\cos\theta(7\tan\theta - 5\cos\theta) = 1$ can be written in the form $a\sin^2\theta + b\sin\theta + c = 0$, where a, b and c are integers to be found. [3]

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(b) Hence solve the equation $\cos 2x(7\tan 2x - 5\cos 2x) = 1$ for $0° < x < 180°$. [3]

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5 The equation of a curve is $y = 2x^2 - \frac{1}{2x} + 3$.

(a) Find the coordinates of the stationary point. [3]

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(b) Determine the nature of the stationary point. [2]

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(c) For positive values of x, determine whether the curve shows a function that is increasing, decreasing or neither. Give a reason for your answer. [2]

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6 A curve passes through the point $\left(\frac{4}{5}, -3\right)$ and is such that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-20}{(5x-3)^2}$.

(a) Find the equation of the curve. [4]

(b) The curve is transformed by a stretch in the x-direction with scale factor $\frac{1}{2}$ followed by a translation of $\begin{pmatrix} 2 \\ 10 \end{pmatrix}$.

Find the equation of the new curve. [3]

7 The first term of an arithmetic progression is 1.5 and the sum of the first ten terms is 127.5 .

(a) Find the common difference. [2]

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(b) Find the sum of all the terms of the arithmetic progression whose values are between 25 and 100. [5]

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8 A circle with equation $x^2 + y^2 - 6x + 2y - 15 = 0$ meets the y-axis at the points A and B. The tangents to the circle at A and B meet at the point P.

Find the coordinates of P. [8]

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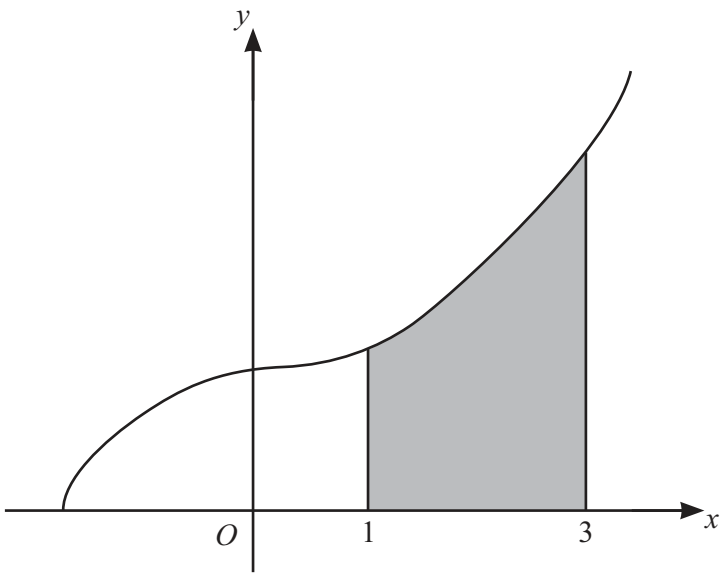

The diagram shows the curve with equation $y = \sqrt{2x^3 + 10}$.

(a) Find the equation of the tangent to the curve at the point where $x = 3$. Give your answer in the form $ax + by + c = 0$ where a, b and c are integers. [5]

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(b) The region shaded in the diagram is enclosed by the curve and the straight lines $x = 1$, $x = 3$ and $y = 0$.

Find the volume of the solid obtained when the shaded region is rotated through 360° about the x-axis. [3]

10 The geometric progression $a_1, a_2, a_3, \ldots$ has first term 2 and common ratio r where $r > 0$.
It is given that $\frac{9}{2}a_5 + 7a_3 = 8$.

(a) Find the value of r. [3]

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(b) Find the sum of the first 20 terms of the geometric progression. Give your answer correct to 4 significant figures. [2]

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(c) Find the sum to infinity of the progression $a_2, a_5, a_8, \ldots$. [3]

11 The function f is defined by $f(x) = 10 + 6x - x^2$ for $x \in \mathbb{R}$.

(a) By completing the square, find the range of f. [3]

The function g is defined by $g(x) = 4x + k$ for $x \in \mathbb{R}$ where k is a constant.

(b) It is given that the graph of $y = g^{-1}f(x)$ meets the graph of $y = g(x)$ at a single point P.

Determine the coordinates of P. [6]

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

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